# Section 2: Error Analysis

This section contains a brief error analysis reading and an error analysis assignment. Together with the lab lectures, recitations, and supplemental readings, this section will provide you with an adequate understanding of basic error and error propagation analysis that will be required in NUCL 325.

***Introduction to Error Analysis***: Brief introduction to error analysis; presents a basic foundation of error analysis principles and includes common error analysis definitions and formulas that may be used in the semester

***Error Analysis Assignment****:*  A homework assignment that introduces and reinforces basic error analysis problems and calculations. The work and answers from this section will be turned in to your TA.

2.1: Introduction to Error Analysis 25

2.2: Error Analysis Assignment 30

## 2.1: Introduction to Error Analysis

*“*‘How *good are the data?’ is the first question put to any experimentalist who draws a conclusion from a set of measurements. The data may become the foundation of a new theory or the undoing of an existing one. They may form a critical test of a structural member in an aircraft wing that must never fail during operation. Before a data set can be used in an engineering or scientific application, its quality must be established…*

*“…Engineers should never simply read a scale or a printout and blindly accept the numbers. They must carefully place realistic tolerances on each of the measured values, and not only should have a doubting mind but also should attempt to quantify their doubts.”*

(Beckwith, Marangoni, & Lienhard, 1993)

Arnold O. Beckman, founder of Beckman Instruments, stated, ‘one thing you learn in science is that there is no *perfect* answer, no *perfect* measure.’ As engineers, it is important to learn to classify the reliability of the results from an experiment or measurement. With this in mind, in publishing experimental results, there needs to be listed error to demonstrate the accuracy of data. These publications can range from journal articles, technical reports for industrial jobs, or even laboratory reports for your professors. Therefore, a complete understanding of error analysis and propagation needs to be understood before beginning any experiment.

**Error** (u) is defined as the difference between the **measured** value (xm), and the **true** (xtrue) value.

In an experiment the *true* value is not known; if it were, there would be no reason to conduct the experiment. Therefore, to accurately estimate error it is important to know the type of error, origin, and how it propagates to analyzed results. With the understanding of the physical process and experimental techniques involved with experimentation, error can be reduced.

#### Types of Error

There are two important concepts that need to be considered in the analysis of measured values. These involve how *accurate* and how *precise* the measurement is. **Accuracy** is a measure of how close the measurement is to the true value. **Precision** is a measure of how well the measurement has been determined, without reference to its agreement with the true value. Precision is also a measure of the reproducibility of a result in a given experiment. An example of these concepts can be seen in Figure 2.1.1. Systematic errors determine the measurement accuracy while random errors determine the measurement precision.



Figure 2.1.: Random vs. systematic error (reproduced from Figure 4.1 of Bevington and Robinson, *Data Reduction and Error Analysis,* 2003).

These two concepts can be considered in the two major categories of error: systematic, or **bias error** and random or **precision error**.

**Bias Error:** These errors occur in a systematic way, and are consistent and repeatable for each measurement. Bias error mainly comes from equipment and scale readings for the measurements being recorded. Two examples of bias errors include zero-offset errors and scale errors. Zero-offset errors occur where a zero measurement was not established properly and the measurement is offset by the same amount in every measurement. Scale errors occurwhere the slope of the output relative to the input is incorrect and will therefore cause a fixed percentage error in all readings (Beckwith, Marangoni, & Lienhard, 1993). Bias errors are typically corrected and prevented by appropriate calibration. They can also be reported in the user manual of equipment or be assumed to be within one unit of the scale on the equipment.

**Precision Error:** This type of error occurs randomly and is present in every data set. They are linked to accuracy in measurement (or lack thereof) and the limitations of the measurement device. Types of precision errors include some human errors, disturbances to the equipment, fluctuations in experimental conditions, etc. Increasing the accuracy and the number of the experimental readings generally reduces precision error in the data.

Bias and precision errors can exist simultaneously in experimental results and can be difficult to identify and separate. In such cases where they are be separate, bias errors and precision errors can be combined to yield the total uncertainty of a measurement as shown in following equation:

where *U* is the total uncertainty of the result, *B* is the bias error and *P* is the precision error.

It is also common to see the result listed with both types of errors specified (i.e. ).

At times when they cannot be separated, remember that the bounds on error represent an *estimation of a likely upper limit* on the error of a result. It is important that all factors contributing to error are considered and estimated to the best of your ability. The error and its bounds give the degree to which numbers can be trusted.

#### Useful Equation in Error Analysis

The following equations refer to those that will need to be considered in the analysis of precision error and total uncertainty of measured results:

Mean value: - most probably single value for the quantity 

Deviation: *d* = = error of a single term from the mean value

Standard deviation of entire population:

Standard deviation of sample:

Standard deviation of mean (SDOM): n = number of measurements taken

Fractional uncertainty: , absolute uncertainty

Percentage uncertainty = Fractional uncertainty x 100%

#### Error Propagation

All experiments essentially consist of two steps: measurement and calculation. Measurement takes place in the laboratory, and calculation takes place in data analysis. When measured values are used in calculation, **the uncertainty of the initial measurement propagates through the calculation to the final result**. The previous section discusses the initial error values for the measurements taken in the laboratory, and this section will take these error values and propagate them to any calculations

Every measured value should be reported with a bias error from equipment. Also, if taking various measurements of the same thing, precision error should be included in the final reported value. When any measured value is used in further calculations and analysis, the error associated with each needs to be propagated through to the result of the calculation. In error propagation, the simplest method of analysis is the use of partial derivatives, yet there are equations that can be used for simple mathematical operations including: addition and subtraction, multiplication and division, and the products of powers. The equations are shown respectively below. All equations presented here are essentially simplifications of the partial derivatives method, which will be discussed after.

#### Simple Propagation Functions:

**Addition and Subtraction:**

Equations:

Knowing *x*, σx, *y* σy, ; *z* from above equation, then σz found through:

**Multiplication by a constant:**

Multiply the uncertainty of the measurement by the same constant

**Multiplication and Division:**

Equations*:*

Knowing *x*, σx, *y* σy, ; *z* from above equation, then σz found through:

**Products of Powers:**

Equations*:*

Knowing *x*, σx, *y* σy, *m, n*; *z* from above equation, then σz found through:

#### Complex Propagation Functions:

If an equation is being performed in analysis that is more complicated than the functions listed previously, the partial derivative of each variable used in the governing equation can be applied as a weighting factor for the uncertainties associated with each variable. With this method, the variable in the experiment with the largest effect on the total uncertainty of the result can be easily distinguished. This identification can be used in future experiments to reduce the uncertainty in the final reported value. Below is the general formation of an equation using partial derivatives of the variables present along with an example.

Function: NOTE: *x, y, w* **MUST** BE INDEPENDENT

Use partial derivatives; first, derive partial derivatives based on function

Then, find σzthrough:

where partial derivatives act as a weighting factor.

**Example:** When calculating the area of a cylinder after measuring the height and radius, the error can be propagated using partial derivatives as shown:

The final error in the area can then be found and represented as shown in the Equations below:

#### Reporting Error/Significant Figures in Error Values

Through measurement and analysis, the discovered results can be reported as a number or a trend.

It is important to remember that simply reporting a value is not enough; it is necessary to report how good the value is. Common practice is to compare experimental results with theory or previous experiments or to report results *with error*. It is also very important to know the *bounds* of error. Imagine the results of your experiment are bounded by some value of the form:

where the bounds of the error, *u*, can be defined as the uncertainty. In this situation, it would be necessary to report the results of the measurement as: xm ± u.

Significant figures can be considered more like a convention, rather than a standard. As you move forward in your careers, your employers will provide you with specific criteria for reporting your results. For this lab, the majority of the guidelines come from John Taylor: *An Introduction to Error Analysis* or from Beckwith *et.al: Mechanical Measurements*.

Guidelines:

*Calculations*: In your calculations, please carry *one more significant figure* than will be reported in your final answer. Doing so will reduce errors associated with rounding.

*Uncertainty*: Your final uncertainty should be on the same order as your least sure parameter. For example, if your result is a function of four parameters, three of which have an uncertainty of ±0.0001, and one of which has an uncertainty of ± 0.1, your final uncertainty must not be better than ± 0.1.

*Combining*: Combining bias and precision error takes place in two steps *before* you perform your error propagation. For example, imagine you measure a parameter three times with an instrument that has a rated uncertainty of ± 0.001. To get the total uncertainty for that parameter, you will first perform statistical analysis on your three readings to find the standard deviation of your parameter, or the *precision* error of your reading. Then you will combine this *precision* error with the *bias* error that comes from the rated uncertainty of your instrument to get the total error, through:

You will then use the *total* error for your parameter (a combination of your precision and bias error) in your error propagation analysis for *each* parameter in your governing equation.

Once you have completed your calculations, you need to make sure that the number of significant figures in your answer does not exceed the significant figures of your uncertainty. If it does you need to adjust your final answer to match.

## 2.2: Error Analysis Assignment

Reading Assignment is posted on Blackboard

1. In your own words, describe the difference between precision and bias error. Give examples.
2. a) A student measures two quantities *a* and *b* with the results *a* = 11.5 ± 0.2 cm and *b* = 25.4 ±0.2 cm. Now, they calculate the product: *q = ab*. Find the final answer, giving both its percentage uncertainty and its absolute uncertainty.

b) Repeat part (a) for the measurements *a* = 10 ± 1 cm and *b* = 27.2 ±0.1 sec

c) Repeat part (a) with *a* = 3.0 ft ± 8% and *b* = 4.0 lb ±2%

1. Compare the design of two bolts based on their tensile strength capabilities. Use the following experimental data. Is there a difference between the two samples at the 95% confidence level? (Reference Example 3.7 from supplemental reading)

|  |  |  |  |
| --- | --- | --- | --- |
| Group | Failure Load | Std. Deviation | Number of Tests |
| A | 30 kN | 2 kN | 21 |
| B | 34 kN | 6 kN | 9 |

1. A brass rod is held under a fixed tensile load and the axial strain in the rod is determined using a strain gauge. Thirty results are obtained under the fixed test conditions, yielding an average strain of ε = 520 -strain. Statistical analysis gives a precision uncertainty of Pε = 21 -strain. The bias uncertainty is estimated from the equipment manual to be Bε = 29 -strain. What is the total uncertainty of the strain?